Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Integral of Log

$$\int \ln x\,dx = x\ln x - x + C.$$

Taylor Series

If the function f is "smooth" at $x = a$, then it can be approximated by the n^{th} degree polynomial

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

Maclaurin Series

A Taylor Series about $x = 0$ is called Maclaurin.

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

Lagrange Error Bound

If $P_n(x)$ is the n_{th} degree Taylor polynomial of $f(x)$ about c and $|f^{(n+1)}(t)| \le M$ for all t between x and c, then

$$|f(x) - P_n(x)| \le \frac{M}{(n+1)!}|x-c|^{n+1}$$

Alternating Series Error Bound

If $S_N = \sum_{k=1}^{N}(-1)^n a_n$ is the N$^{\text{th}}$ partial sum of a convergent alternating series, then

$$|S_\infty - S_N| \le |a_{N+1}|$$

Euler's Method

If given that $\frac{dy}{dx} = f(x,y)$ and that the solution passes through (x_0, y_0),

$$y(x_0) = y_0$$

$$\vdots$$

$$y(x_n) = y(x_{n-1}) + f(x_{n-1}, y_{n-1}) \cdot \Delta x$$

In other words:

$$x_{\text{new}} = x_{\text{old}} + \Delta x$$

$$y_{\text{new}} = y_{\text{old}} + \left.\frac{dy}{dx}\right|_{(x_{\text{old}}, y_{\text{old}})} \cdot \Delta x$$

Ratio Test

The series $\sum_{k=0}^{\infty} a_k$ converges if

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| < 1.$$

If limit equals 1, you know nothing.

Polar Curves

For a polar curve $r(\theta)$, the **Area** inside a "leaf" is

$$\int_{\theta 1}^{\theta 2} \frac{1}{2}[r(\theta)]^2 d\theta,$$

where $\theta 1$ and $\theta 2$ are the "first" two times that $r = 0$.

The **slope** of $r(\theta)$ at a given θ is

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta}[r(\theta)\sin\theta]}{\frac{d}{d\theta}[r(\theta)\cos\theta]}$$

l'Hopital's Rule

If $\frac{f(a)}{g(a)} = \frac{0}{0}$ or $= \frac{\infty}{\infty}$, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$.