AP CALCULUS
Stuff you **MUST** Know Cold

Curve sketching and analysis

$y = f(x)$ must be continuous at each:

critical point: $\frac{dy}{dx} = 0$ or undefined.

local minimum :
- $\frac{dy}{dx}$ goes (-,0,+) or (-,und,+)
- or $\frac{d^2y}{dx^2} > 0$.

local maximum :
- $\frac{dy}{dx}$ goes (+,0,-) or (+,und,-)
- or $\frac{d^2y}{dx^2} < 0$.

pt of inflection : concavity changes.
- $\frac{d^2y}{dx^2}$ goes (+,0,-),(-,0,+), (+,und,-), or (-,und,+)

Basic Derivatives

$$\frac{d}{dx}(x^n) = nx^{n-1}$$
$$\frac{d}{dx}(\sin x) = \cos x$$
$$\frac{d}{dx}(\cos x) = -\sin x$$
$$\frac{d}{dx}(\tan x) = \sec^2 x$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x$$
$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$
$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$
$$\frac{d}{dx}(e^x) = e^x$$

More Derivatives

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$
$$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$
$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}(a^x) = a^x \ln a$$
$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

Differentiation Rules

Chain Rule
$$\frac{d}{dx}[f(u)] = f'(u)\frac{du}{dx}$$
$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Product Rule
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$$

Quotient Rule
$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$$

"PLUS A CONSTANT"

The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$
where $F'(x) = f(x)$.

Corollary to FTC

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)dt = f(b(x))\,b'(x) - f(a(x))\,a'(x)$$

Intermediate Value Theorem

If the function $f(x)$ is continuous on $[a,b]$, then for any number c between $f(a)$ and $f(b)$, there exists a number d in the open interval (a,b) such that $f(d) = c$.

Rolle's Theorem

If the function $f(x)$ is continuous on $[a,b]$, the first derivative exist on the interval (a,b), and $f(a) = f(b)$; then there exists a number $x = c$ on (a,b) such that
$$f'(c) = 0.$$

Mean Value Theorem

If the function $f(x)$ is continuous on $[a,b]$, and the first derivative exists on the interval (a,b), then there exists a number $x = c$ on (a,b) such that
$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

Theorem of the Mean Value

If the function $f(x)$ is continuous on $[a,b]$ and the first derivative exist on the interval (a,b), then there exists a number $x = c$ on (a,b) such that
$$f(c) = \frac{\int_a^b f(x)dx}{(b-a)}.$$
This value $f(c)$ is the "average value" of the function on the interval $[a,b]$.

Trapezoidal Rule

$$\int_a^b f(x)dx = \frac{b-a}{2n}[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

Solids of Revolution and friends

Disk Method
$$V = \pi\int_a^b [R(x)]^2\,dx$$

Washer Method
$$V = \pi\int_a^b \left([R(x)]^2 - [r(x)]^2\right)dx$$

Shell Method(no longer on AP)
$$V = 2\pi\int_a^b r(x)h(x)dx$$

ArcLength
$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

Surface of revolution (No longer on AP)
$$S = 2\pi\int_a^b r(x)\sqrt{1 + [f'(x)]^2}\,dx$$

Distance, velocity and acceleration

velocity $= \frac{d}{dt}$(position).

acceleration $= \frac{d}{dt}$(velocity).

velocity vector $= \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$.

speed $= |v| = \sqrt{(x')^2 + (y')^2}$.

$$\text{Distance} = \int_{\text{initial time}}^{\text{final time}} |v|dt = \int_{t_0}^{t_f} \sqrt{(x')^2 + (y')^2}\,dt$$

average velocity $= \frac{\text{final position} - \text{initial position}}{\text{total time}}$.

average velocity $= \frac{s(b) - s(a)}{b - a}$